

Pion charge form factor and constraints from space-time translations

Bertrand Desplanques^{1*}, Yubing Dong^{2,3†}

¹ Laboratoire de Physique Subatomique et Cosmologie,
Université Joseph Fourier Grenoble 1, CNRS/IN2P3, INPG, France

² Institute of High Energy Physics, Chinese Academy of Sciences,
Beijing 100049, P. R. China

³ Theoretical Physics Center for Science Facilities (TPCSF), CAS,
Beijing 100049, P. R. China

February 18, 2010

Abstract

The role of Poincaré covariant space-time translations is investigated in the case of a relativistic quantum mechanics approach to the pion charge form factor. It is shown that the related constraints are generally inconsistent with the assumption of a single-particle current, which is most often referred to. The only exception is the front-form approach with $q^+ = 0$. How accounting for the related constraints, as well as restoring the equivalence of different RQM approaches in estimating form factors, is discussed. Some extensions of this work and, in particular, the relationship with a dispersion-relation approach, are presented. Conclusions relative to the underlying dynamics are given.

Key words: form factors, pion, relativist, covariance, translations

PACS: 12.39.Ki, 13.40.Gp, 14.40.Aq

1 Introduction

It is a usual claim that the study of form factors of hadronic systems should *a priori* provide information on the underlying dynamics. Examination of estimates in the framework of relativistic quantum mecha-

tics (RQM) nevertheless shows a strong sensitivity to the choice of the form used to implement relativity [1], especially for the pion charge form factor [2, 3]. The dependence on the form and its associated construction of the Poincaré algebra in instant, front and point ones [4, 5] results from an incomplete

**E-mail address:* desplanq@lpsc.in2p3.fr

†*E-mail address:* dongyb@ihep.ac.cn

calculation. It is expected that it should disappear by accounting for many-particle terms in the current besides a single-particle one that is usually retained [6]. In absence of the many-particle terms, one can only hope that one approach is better than other ones but its choice may reflect some own prejudice, necessarily subjective.

Recently, an objective argument, based on properties of currents under Poincaré space-time translations, was presented, allowing one to discriminate between different RQM implementations [7, 8]. Invariance under space-time translations implies the expected energy-momentum conservation but this only represents a part of properties that can be ascribed to these transformations in the RQM framework. As shown by Lev [9], relations involving the commutator of the space-time translation operator, P^μ , with the currents, which stem from Poincaré covariance, have also to be verified. Their consideration [7, 8] suggested a way to account indirectly for the above many-particle currents for a scalar system consisting of scalar constituents [10, 11]. The procedure tends to restore the equality of the squared momentum transferred to the constituents and to the whole system, which is *a priori* violated in incomplete RQM approaches but holds in field-theory ones. This work is extended here to a physical system, the pion, which represents one of the simplest hadron and, moreover, has been the object of extensive studies (see [12] for references).

The plan of the paper is as follows. We first review properties that currents should fulfill under space-time translations and pay a particular attention to constraints they imply and go beyond the energy-momentum conservation. In the following part, we show how we account for these constraints. Results for the pion charge form factor in different forms are then presented together with the result obtained when the above constraints are accounted for. In the last

part, we make various comments in relation with possible extensions. A conclusion summarizes the main results and the consequences that a comparison with experiment suggests for the solution of the mass operator used in our calculations.

2 Covariant space-time translations: new insight and constraints

Properties under Poincaré covariant space-time translations imply the following transformations of currents:

$$e^{iP \cdot a} J^\nu(x) (S(x)) e^{-iP \cdot a} = J^\nu(x+a) (S(x+a)), \quad (1)$$

where P^μ represents the 4-momentum operator. The quantities $J^\nu(x)$ and $S(x)$ respectively refer to 4-vector and scalar currents. When the matrix element of the above relations for $a = -x$ is taken between eigenstates of P^μ , one obtains:

$$\begin{aligned} &< i | J^\nu(x) \text{ (or } S(x)) | f > \\ &= e^{i(P_i - P_f) \cdot x} < i | J^\nu(0) \text{ (or } S(0)) | f > . \end{aligned} \quad (2)$$

Together with the function $e^{iq \cdot x}$ describing the interaction with an external probe carrying momentum q^μ , and assuming space-time translation invariance, one obtains the standard energy-momentum conservation relation:

$$(P_f - P_i)^\mu = q^\mu . \quad (3)$$

In calculating the matrix element of Eq. (2), and probably for simplicity, it is generally assumed that $J(0)^\mu$ (or $S(0)$) is described by a single-particle operator. Until recently however, it was not checked whether this assumption is consistent with further constraints that stem from Eq. (1) and were proposed by Lev [9]. Particular relations of

relevance here are the following ones:

$$\begin{aligned} [P_\mu, [P^\mu, J^\nu(x)]] &= -\partial_\mu \partial^\mu J^\nu(x), \\ [P_\mu, [P^\mu, S(x)]] &= -\partial_\mu \partial^\mu S(x). \end{aligned} \quad (4)$$

After factorizing the x dependence as in Eq. (2), and taking the matrix element of the current, assuming temporarily it is a single-particle one, one should get the following relations:

$$\begin{aligned} &< |q^2 J^\nu(0) \text{ (or } S(0)) | > \\ &= < |(p_i - p_f)^2 J^\nu(0) \text{ (or } S(0)) | >, \end{aligned} \quad (5)$$

where q^2 represents the squared momentum transferred to the system and $(p_i - p_f)^2$ the one transferred to the constituents. Checking the relations, it is found that they are violated in all cases with one exception: the front-form approach with the momentum configuration $q^+ = 0$. The violation of the relations therefore shows that the assumption of a single-particle current is not valid in the corresponding approaches, indicating that the current should then contain many-particle terms. One can hope that accounting for their contributions would restore the equivalence of different approaches in calculating form factors [6]. However, calculating the contribution of many-particle terms is quite tedious and this has been done only in a limited number of cases [13, 12]. Moreover, if they have the effect of restoring the equivalence with other approaches, they should occur at all orders in the interaction, which is apparently hopeless.

3 Implementation of the constraints

In order to account for the extra contributions, we observe that the current should in one way or another keep the structure of a single-particle one as far as this is the case

for the front form with $q^+ = 0$, which fulfills the constraints. Moreover, comparing different approaches, we notice that their expressions differ in the coefficient multiplying the momentum transfer q^μ and that the difference implies interaction effects that are here or there depending on the approach. These observations suggest to modify the factor of the momentum transfer by a coefficient α , so that to fulfill Eq. (5). We thus obtain the equation:

$$\begin{aligned} q^2 &= "[(P_i - P_f)^2 + 2 (\Delta_i - \Delta_f) (P_i - P_f) \cdot \xi \\ &\quad + (\Delta_i - \Delta_f)^2 \xi^2]" \\ &= \alpha^2 q^2 - 2\alpha "(\Delta_i - \Delta_f)" q \cdot \xi \\ &\quad + "(\Delta_i - \Delta_f)^2" \xi^2, \end{aligned} \quad (6)$$

where ξ^μ represents the orientation of the hyperplane on which physics is described and Δ holds for an interaction effect. It is immediately seen that, for the front-form case with $q^+ = 0$, the above equation is satisfied with $\alpha = 1$ as $\xi^2 = 0$ and $\xi \cdot q = 0$. In this case, the equality of the squared momentum transferred to the system and to the constituents, Eq. (5), is trivially fulfilled. In the other cases, one has to take into account the modification of the calculation given by the coefficient α , which is solution of Eq. (6).

4 Some results

The detail of the implementation of the constraints stemming from space-time translations has been given in Ref. [14] for a scalar system like the pion one consisting of two spin-1/2 particles. Recovering the equivalence of different approaches may require a particular current but, in the case of the charge form factor, the current so obtained is often the one that is expected, probably because it fulfills minimal requirements such as the existence of an underlying conserved current or the invariance of the

charge (unaffected by the constraints) under boosts. For the wave function, we use the one obtained in an independent work [12] aimed to study the asymptotic behavior of the form factor in RQM approaches. The corresponding mass operator involves both a confinement and an instantaneous one-gluon exchange interaction.

Results are presented in Fig. 1 for various approaches (see Refs. [7, 8] for definitions and further details). The left panel, which involves low Q^2 , is sensitive to the squared charge radius while the right panel covers the remaining range of Q^2 where data are available.

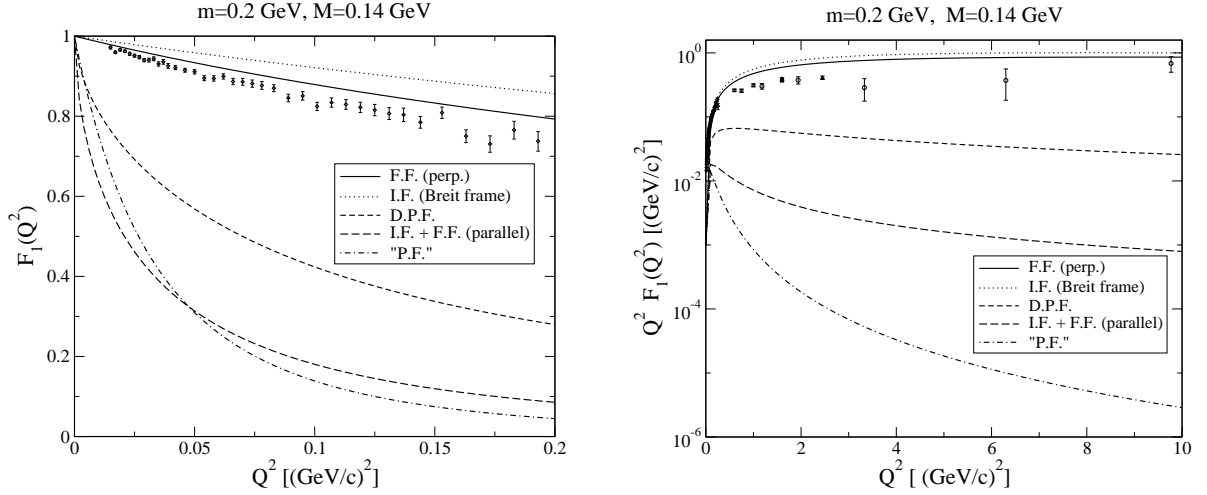


Figure 1: Pion charge form factor in different forms and effect of the constraints related to the covariance properties of the current under space-time translations: left panel for the low- Q^2 range and right panel for the intermediate- Q^2 range (see text for references about further details and some definitions). The dot, short-dash, long-dash, dot-dash and continuous curves represent results without the effect of constraints discussed here. When this effect is included, all curves become identical to the front-form one with $q^+ = 0$, which is unchanged (continuous curve denoted F.F. (perp.)).

Examination of the figure shows a considerable discrepancy between different approaches, which is very similar to the spinless-constituent case [15, 7, 8]. At low Q^2 , the discrepancy exhibits in some cases a dependence of the squared radius on the inverse of the squared pion mass. It is noticed that the Lorentz invariance of the form factor obtained in an earlier point-form approach ("P.F.") [16] or in a point-form approach inspired from Dirac's one (D.P.F.) [17] does not guarantee a better result. When the effect of the implementation of the constraints related to space-time trans-

lations is accounted for, all form factors become identical to the front-form one with $q^+ = 0$, which was fulfilling the constraints from the start. Hence, the full restoration of properties related to the Poincaré covariance of these transformations is essential to obtain reliable results.

5 Further developments

While showing numerically that expressions of the pion form factor in different forms could give the same results after accounting

for constraints related to space-time translations, we wondered what could be the common expression behind this important property. Following different works in the scalar constituent or in the spin-1/2 cases [18, 19, 8], it is found that this expression could be identified to the one based on a s-channel dispersion-relation approach, which is explicitly Lorentz invariant and, moreover, fulfills constraints related to space-time translations. This expression, which is not well known, reads:

$$F_1(Q^2) = \frac{1}{N} \int d\bar{s} d\left(\frac{s_i - s_f}{Q}\right) \phi(s_i) \phi(s_f) \times \frac{2\sqrt{s_i s_f} \theta(\dots)}{D\sqrt{D}}, \quad (7)$$

where the variables and various quantities are defined in Ref. [8]. Obtaining this result in each form supposes that the corresponding implementation of relativity represented by the Bakamjian-Thomas construction of the Poincaré algebra in the instant form [4] and its generalizations in the other cases [5] has been consistently performed. It also supposes a non-trivial change of variables. We notice that the above expression confirms the one given in Ref. [18] but disagrees with the one given in Ref. [19]. For this last work, the discrepancy factor, $(s_i + s_f + Q^2)/(2\sqrt{s_i s_f})$, is the same as for the scalar-constituent case [8].

We mentioned that accounting for constraints related to space-time translations was amounting to implicitly consider the contribution of many-particles currents. These currents only represent a minimal subset, which is required to restore some symmetry properties. In the present case of the pion charge form factor, they do not allow one to reproduce its asymptotic expression. This one supposes to consider specific two-particle currents within the RQM approach [12].

It is not rare that the breaking of some symmetry can provide unexpected results,

at the limit of paradoxes. The variation of the charge radius with the inverse of the pion mass obtained in some cases is one of them (more binding produces a larger radius!). As shown here, accounting for constraints from Poincaré covariant translations corrects for this surprising result.

6 Conclusion

We have considered the role of constraints relative to space-time translations on the estimate of the pion charge form factor. Accounting for these constraints amounts to restore the equality of the squared momentum transferred to the constituents and to the pion, which is violated in the simplest RQM calculation but is trivially fulfilled in field theory. As these constraints stem from a covariant transformation of currents under space-time translations, accounting for them represents a necessary ingredient of a covariant calculation of form factors. A complementary insight is the following one. In RQM approaches, changing the underlying hypersurface for another one implies interaction effects. One is not therefore surprised that the simplest calculation of elastic form factors depends on its choice. Accounting for the constraints discussed in this work amounts to consider further interaction effects that remove the differences implied by this choice, as expected from a fully Poincaré covariant calculation.

Considering the front-form results with $q^+ = 0$ as representative of the results obtained in different approaches after accounting for constraints related to space-time translations, it appears that the calculated form factor tends to overestimate the measured one. As there was no optimisation of the estimate, one can think that a better value of the pion decay constant, f_π , could help to explain the measurements in the low- Q^2 range. The statement is based on

the relation $r_\pi^2 = 3/(4\pi^2 f_\pi^2)$ and a calculated value of f_π which overestimates the measured one (106 and 93 MeV respectively). A better value of f_π could be obtained by diminishing the quark mass or by reducing the weight of high-momentum components in the solution of the mass operator that was used. This second alternative would have the advantage to also reduce the discre-

pancy in the intermediate- Q^2 range.

Acknowledgments This work is partly supported by the National Sciences Foundations of China under grants No. 10775148,10975146 (Y.B.). The author is also grateful to the CAS for grant No KJCX3-SYW-N2. B.D. is very grateful to IHEP for offering hospitality after the conference.

References

-
- [1] P.A.M. Dirac, Rev. Mod. Phys., 1949, **21**: 392
 - [2] A. Amghar, B. Desplanques, L. Theußl, Phys. Lett. B, 2003, **574**: 201
 - [3] Jun He, B. Julia-Diaz, Yu-bing Dong, Phys. Lett. B, 2004, **602**: 212
 - [4] B. Bakamjian, L.H. Thomas, Phys. Rev., 1953, **92**: 1300
 - [5] B. Keister, W. Polyzou, Adv. Nucl. Phys., 1991, **20**: 225
 - [6] S.N. Sokolov, A.N. Shatnii, Theor. Math. Phys., 1978, **37**: 1029
 - [7] B. Desplanques, [nucl-th/0407074].
 - [8] B. Desplanques, Yu-bing Dong, Eur. Phys. J. A, 2008, **37**: 33
 - [9] F.M. Lev, Rivista del Nuovo Cimento, 1993, **16**: 1
 - [10] G.C. Wick, Phys. Rev., 1954, **96**: 1124
 - [11] R.E. Cutkosky, Phys. Rev., 1954, **96**: 1135
 - [12] B. Desplanques, Eur. Phys. J. A, 2009, **42**: 219
 - [13] B. Desplanques, L. Theußl, Eur. Phys. J. A, 2004, **21**: 93
 - [14] B. Desplanques, Yu-bing Dong, arXiv:0907.2835 [nucl-th]
 - [15] A. Amghar, B. Desplanques, L. Theußl, Nucl. Phys., 2003, **A 714**: 213
 - [16] B. Bakamjian, Phys. Rev., 1961, **121**: 1849
 - [17] B. Desplanques, Nucl. Phys., 2005, **A 748**: 139
 - [18] D. Melikhov, Eur. Phys. J. direct, 2002, **C4**: 2 [hep-ph/0110087]
 - [19] A.F. Krutov, V.E. Troitsky, Phys. Rev., 2002, **C 65**: 045501